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Laminar flow through parallel and uniformly porous walls of different permeability

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The problem of laminar flow through parallel and uniformly porous walls of different permeability is investigated. Adopting the velocity components in terms of unknown functions in such a way that the boundary conditions become independent of Reynolds number of the cross-flow, we have obtained the velocity distribution and the stream function for small Reynolds number.

NOMENCLATURE

(x, y)	Co-ordinate system x-axis along the lower plate and y is measured normal to it
$(\bar{x}, \bar{y})$	Non-dimensional co-ordinates
(u, v)	The velocity components of the fluid particle along the axes.
$(\bar{u}, \bar{v})$	Non-dimensional form of velocity components
ρ	Density of the fluid
ν	Kinematic coefficient of viscosity.
V_1	Injection velocity at the lower plate along y-axis.
V_2	Suction velocity at the upper plate along y-axis.
y_0	The distance between two plates.
R_1	Injection Reynolds number $(= V_1 y_0/\nu)$
R_2	Suction Reynolds number $(= V_2 y_0/\nu)$.
K	Non-dimensional pressure gradient $(= -\partial\bar{p}/\partial\bar{x})$.
P_x, P_y	Pressure drop in the $\bar{x}$ and $\bar{y}$ direction.
c_f	Coefficient of skin friction.

1. INTRODUCTION

Berman (1953) has investigated the problem of steady laminar flow of a viscous incompressible fluid through a porous channel and he has found a solution for small Reynolds number assuming the normal velocities at the walls to be equal Verma & Bansal (1966) have discussed the flow of viscous incompressible fluid between parallel plates, one in uniform motion and the other at rest with uniform suction at the stationary plate. They have pointed out that for large values of x, an adverse pressure gradient is developed which causes a back flow. Terrill

& Shrestha (1965) have investigated the problem of laminar flow through parallel and uniformly porous walls of different permeability. They have obtained a series solution for small Reynolds numbers in section (5) of their paper. They have expanded the unknown functions which occur in the non-linear ordinary differential equation in a power series for small Reynolds number R_2 and pointed that their f_r's and C_r's are independental of R_2 while, the boundary conditions for determining the f_r's involve α_2 which is equal to $(1-R_1/R_2)$. Thus in the final solution they have obtained expressions for f_r's containing powers of α_2 which involves R_2, due to boundary conditions. This is contrary to their initial assumption for f_r's.

In this paper we have considered the same problem again, keeping in view the above mentioned drawbacks Adopting the velocity components in terms of unknown functions in such a way that the boundary conditions become independent of Reynolds number of the cross-flow, we have obtained the velocity distribution and the stream function for small Reynolds number. The stream lines are drawn for the case of suction at both walls and for suction at one wall and injection at the other wall

2. Formulation of the Problem

The equations of motion and continuity for the problem under consideration are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \qquad \dots (1)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \qquad \dots (2)$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \dots (3)$$

Since there is uniform suction and injection, $\frac{\partial v}{\partial x}=0$; and from equation (3), we have $\frac{\partial^2 u}{\partial x^2}=0$.

Thus equations (1) to (3) reduces to the following :

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}, \qquad \dots (4)$$

$$v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial y^2}, \qquad \dots (5)$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \qquad \ldots (6)$$

Now let us introduce the following non-dimensional quantities :

$$\left.\begin{aligned} \bar{u}=\frac{u}{(\nu/y_0)},\ \bar{v}=\frac{v}{(\nu/y_0)},\ \bar{x}=\frac{x}{y_0},\ \bar{y}=\frac{y}{y_0},\\ \bar{p}=\frac{p}{\rho(\nu/y_0)^2},\ R_1=\frac{V_1y_0}{\nu}\text{ and }R_2=\frac{V_2y_0}{\nu}. \end{aligned}\right\} \qquad \ldots (7)$$

Hence equations (4) to (6) in non-dimensional form can be written as follows

$$\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=-\frac{\partial \bar{p}}{\partial \bar{x}}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \qquad \ldots (8)$$

$$v\frac{\partial \bar{v}}{\partial \bar{y}}=-\frac{\partial \bar{p}}{\partial \bar{y}}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}, \qquad \ldots (9)$$

and

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0. \qquad \ldots (10)$$

3. Boundary Conditions

We assume the fluid to be confined between two parallel plates at $\bar{y}=0$ and $\bar{y}=1$. The fluid is moving under the application of constant non-dimensional pressure gradient K. Since both the plates are at rest and there is injection at lower plate and suction at the upper plate, the boundary conditions in non-dimensional form are :

$$\left.\begin{aligned} \bar{u}=0,\quad \bar{v}=R_2\quad \text{at}\quad \bar{y}=1,\\ \text{and}\qquad \bar{u}=0,\quad \bar{v}=R_1\quad \text{at}\quad \bar{y}=0. \end{aligned}\right\} \qquad \ldots (11)$$

Also

$$-\frac{\partial \bar{p}}{\partial \bar{x}}=K \text{ at } \bar{x}=0. \qquad \ldots (12)$$

4. Method of Solution

The form of the normal components of the velocity is assumed in such a way that the boundary conditions (11) become independent of R_1 and R_2 Hence, let us assume

$$\bar{v}=(R_1-R_2)f(\bar{y})+R_2. \qquad \ldots (13)$$

Therefore, from equation (10), we have

$$\bar{u} = (R_2-R_1)\,\bar{x}f'(\bar{y})+F(\bar{y}), \qquad \dots (14)$$

where $f(\bar{y})$ and $F(\bar{y})$ are the unknown functions to be determined

Also from equation (9) we have

$$\bar{p} = (R_1-R_2)f'-\tfrac{1}{2}(R_1-R_2)^2f^2-R_2(R_1-R_2)f+p_0(\bar{x}), \qquad \dots (15)$$

where $p_0(\bar{x})$ is the unknown function of $\bar{x}$.

From equations (13) to (15) and (8) we have

$$[(R_2-R_1)\bar{x}f'+F](R_2-R_1)f'+[(R_1-R_2)f+R_2]$$

$$[(R_2-R_1)\bar{x}f''+F']-[(R_2-R_1)\bar{x}f'''+F''] = -\frac{\partial p_0}{\partial \bar{x}},$$

$$= (R_1-R_2)\bar{x}A+B, \quad \text{(say)} \qquad \dots (16)$$

where A and B are constants to be determined

From equations (12) and (16), we have

$$B = -K.$$

Thus from equation (16) the following ordinary differential equations

$$(R_1-R_2)[f'^2-ff'']-R_2f''+f''' = A, \qquad \dots (17)$$

and

$$(R_1-R_2)[Ff'-fF']-R_2F'+F'' = -K, \qquad \dots (18)$$

are obtained.

The boundary conditions from equations (11), (13) and (14) are

$$\left.\begin{aligned} &f(1) = 0 = f'(1), \quad F(1) = 0, \\ \text{and} \quad &f(0) = 1, \quad f'(0) = 0, \quad F(0) = 0. \end{aligned}\right\} \qquad \dots (19)$$

5. Series Solution for Small Values of R_1 and R_2

For the solution of non-linear ordinary differential equations (17) and (18) let us assume that the unknown functions $f(\bar{y})$, $F(\bar{y})$ and the constant A can be expressed by a power series in R_1 and R_2 as follows :

$$f(\bar{y}) = \Sigma_{r,s}(R_2)^r(R_1)^s f_{rs}(\bar{y}), \qquad \dots (20)$$

$$F(\bar{y}) = \Sigma_{r,s}(R_2)^r(R_1)^s F_{rs}(\bar{y}), \qquad \dots (21)$$

and

$$A = \Sigma_{r,s}(R_2)^r(R_1)^s A_{rs}, \qquad \dots (22)$$

where f_{rs}, F_{rs} and A_{rs} are independent of R_1 and R_2.

From equations (17), (20) and (22), we have

$$f_{00}''' = A_{00}, \quad \dots (23)$$

$$f_{01}''' = A_{01}+f_{00}f_{00}''-f_{00}'^2, \quad \dots (24)$$

....

$$f_{10}''' = A_{10}+f_{00}'^2-f_{00}f_{00}''+f_{00}'', \quad \dots (25)$$

....

The boundary conditions (19) with the aid of (20) are

$$\left.\begin{aligned} &f_{rs}(1) = 0 = f'_{rs}(1) \quad \text{for} \quad r \geqslant 0 \quad \text{and} \quad s \geqslant 0, \\ &f_{00}(0) = 1;\ f_{rs}(0) = 0 \text{ for } r \geqslant 0 \text{ and } s \geqslant 0 \text{ except } r = s = 0, \\ &f'_{rs}(0) = 0 \text{ for } r \geqslant 0 \text{ and } s \geqslant 0, \\ \text{and} \quad &F_{rs}(0) = 0 = F_{rs}(1) \text{ for } r \geqslant 0 \quad \text{and } s \geqslant 0. \end{aligned}\right\} \quad \dots (26)$$

The solutions of equations (23) to (25) with boundary conditions (26) are

$$f_{00} = 2\bar{y}^3-3\bar{y}^2+1, \quad \dots (27)$$

$$f_{01} = -\frac{2}{35}\bar{y}^7+\frac{1}{5}\bar{y}^6-\frac{3}{10}\bar{y}^5+\tfrac{1}{2}\bar{y}^4-\frac{43}{70}\bar{y}^3+\frac{19}{70}\bar{y}^2, \quad \dots (28)$$

$$f_{10} = \frac{2}{35}\bar{y}^7-\frac{1}{5}\bar{y}^6+\frac{3}{10}\bar{y}^5-\frac{27}{70}\bar{y}^3+\frac{8}{35}\bar{y}^2, \quad \dots (29)$$

and

$$A = 12+\frac{81}{35}R_1-\frac{81}{35}R_2. \quad \dots (30)$$

From equations (18), (20) to (22), we have

$$F_{00}'' = -K, \quad \dots (31)$$

$$F_{01}'' = F_{00}'f_{00}-F_{00}f_{00}', \quad \dots (32)$$

$$F_{02}'' = f_{00}F_{01}'+f_{01}F_{00}'-F_{00}f_{01}'-F_{01}f_{00}', \quad \dots (33)$$

....,

$$F_{10}'' = F_{00}f_{00}'-f_{00}F_{00}'+F_{00}', \quad \dots (34)$$

$$F_{11}'' = F_{00}(f_{01}'-f_{10}')+f_{00}(F_{10}'-F_{01}')+f_{00}'(F_{01}-F_{10}) +F_{00}'(f_{10}-f_{01})+F_{01}', \quad \dots (35)$$

....,

and

$$F_{20}'' = F_{00}f_{10}'+F_{10}f_{00}'-F_{10}'f_{00}-f_{10}F_{00}'+F_{10}', \quad \dots (36)$$

...., The solutions of equations (30) to (35) with the boundary conditions (26)

$$F_{00} = \frac{K}{2}(\bar{y}-\bar{y}^2), \qquad \dots (37)$$

$$F_{01} = \frac{K}{120}[4\bar{y}^6-12\bar{y}^5+15\bar{y}^4-20\bar{y}^3+30\bar{y}^2-17\bar{y}], \qquad \dots (38)$$

$$F_{02} = K[0{\cdot}0006\bar{y}^{10}-0{\cdot}0031\bar{y}^9+0{\cdot}0035\bar{y}^8+0{\cdot}0131\bar{y}^7-0{\cdot}0519\bar{y}^6$$
$$+0{\cdot}0840\bar{y}^5-0{\cdot}0708\bar{y}^4+0{\cdot}0833\bar{y}^3-0{\cdot}0708\bar{y}^2+0{\cdot}0121\bar{y}], \qquad \dots (39)$$

$$F_{10} = \frac{K}{120}[-4\bar{y}^6+12\bar{y}^5-15\bar{y}^4+7\bar{y}], \qquad \dots (40)$$

$$F_{11} = K[-0{\cdot}0012\bar{y}^{10}+0{\cdot}0063\bar{y}^9-0{\cdot}0071\bar{y}^8-0{\cdot}0095\bar{y}^7+0{\cdot}0454\bar{y}^6$$
$$-0{\cdot}0764\bar{y}^5+0{\cdot}0517\bar{y}^4+0{\cdot}0291\bar{y}^2-0{\cdot}0383\bar{y}], \qquad \dots (41)$$

and

$$F_{20} = K[0{\cdot}0006\bar{y}^{10}-0{\cdot}0031\bar{y}^9+0{\cdot}0035\bar{y}^8-0{\cdot}035\bar{y}^7+$$
$$+0{\cdot}0064\bar{y}^6-0{\cdot}0076\bar{y}^5-0{\cdot}005\bar{y}^4+0{\cdot}0087\bar{y}] \qquad \dots (42)$$

Thus from equations (27) to (29) and (37) to (42) for the second-order perturbation solution the velocity components are

$$\bar{u} = (R_2-R_1)\bar{x}\Big[(6\bar{y}^2-6\bar{y})+R_1\Big(-\frac{2}{5}\bar{y}^6+\frac{6}{5}\bar{y}^5-\frac{3}{2}\bar{y}^4+2\bar{y}^3-\frac{129}{70}\bar{y}^2+$$
$$+\frac{19}{35}\bar{y}\Big)+R_2\Big(\frac{2}{5}\bar{y}^6-\frac{6}{5}\bar{y}^5+\frac{3}{2}\bar{y}^4-\frac{81}{70}\bar{y}^2+\frac{16}{35}\bar{y}\Big)\Big]$$
$$+\frac{K}{2}(\bar{y}-\bar{y}^2)+\frac{R_1K}{120}(4\bar{y}^6-12\bar{y}^5+15\bar{y}^4-20\bar{y}^3+30\bar{y}^2-17\bar{y})$$
$$+R_1^2K(0{\cdot}0006\bar{y}^{10}-0{\cdot}0031\bar{y}^9+0{\cdot}0035\bar{y}^8+0{\cdot}0131\bar{y}^7)-0{\cdot}0519\bar{y}^6$$
$$+0{\cdot}084\,\bar{y}^5-0{\cdot}0708\bar{y}^4+0{\cdot}0833\bar{y}^3-0{\cdot}0708\bar{y}^2+0{\cdot}0121\bar{y})$$
$$+R_2\Big[\frac{K}{120}(-4\bar{y}^6+12\bar{y}^5-15\bar{y}^4+7\bar{y})+R_1K(-0{\cdot}0012\bar{y}^{10}$$
$$+0{\cdot}0063\bar{y}^9-0{\cdot}0071\bar{y}^8-0{\cdot}0095\bar{y}^7+0{\cdot}0454\bar{y}^6-0{\cdot}0764\bar{y}^5$$
$$+0{\cdot}0517\bar{y}^4+0{\cdot}0291\bar{y}^2-0{\cdot}0383\bar{y})\Big]+R_2^2K(0{\cdot}0006\bar{y}^{10}-0{\cdot}0031\bar{y}^9$$
$$+0{\cdot}0035\bar{y}^8-0{\cdot}0035\bar{y}^7+0{\cdot}0064\bar{y}^6-0{\cdot}0076\bar{y}^5-0{\cdot}005\bar{y}^4+0{\cdot}0087\bar{y}), \qquad \dots (43)$$

and

$$\bar{v} = (R_1-R_2)\Big[(2\bar{y}^3-3\bar{y}^2+1)+R_1\Big(-\frac{2}{35}\bar{y}^7+\frac{1}{5}\bar{y}^6$$

$$-\frac{3}{10}\bar{y}^5+\tfrac{1}{2}\bar{y}^4-\frac{43}{70}\bar{y}^3+\frac{19}{70}\bar{y}^2\Big)+R_2\Big(\frac{2}{35}\bar{y}^7$$

$$-\frac{1}{5}\bar{y}^6+\frac{3}{10}\bar{y}^5-\frac{27}{70}\bar{y}^3+\frac{8}{35}\bar{y}^2\Big)\Big]+R_2\,. \quad \ldots (44)$$

6. Stream Function of the Flow

Let us introduce a stream function ψ such that

$$\bar{u} = \frac{\partial\psi}{\partial\bar{y}}, \quad \ldots (45)$$

and

$$\bar{v} = -\frac{\partial\psi}{\partial\bar{x}} \quad \ldots (46)$$

Also, we have

$$d\psi = \frac{\partial\psi}{\partial\bar{x}}\,d\bar{x}+\frac{\partial\psi}{\partial\bar{y}}\,dy. \quad \ldots (47)$$

Hence from equations (43) to (47), we have

$$\psi = \Big[(R_2-R_1)\Big\{(2\bar{y}^3-3\bar{y}^2+1)+R_1\Big(-\frac{2}{35}\bar{y}^7+\frac{1}{5}\bar{y}^6-\frac{3}{10}\bar{y}^5+\frac{1}{2}\bar{y}^4-\frac{43}{70}\bar{y}^3$$

$$+\frac{19}{70}\bar{y}^2\Big)+R_2\Big(\frac{2}{35}\bar{y}^7-\frac{1}{5}\bar{y}^6+\frac{3}{10}\bar{y}^5-\frac{27}{70}\bar{y}^3+\frac{8}{35}\bar{y}^2\Big)\Big\}-R_2\Big]\,\bar{x}$$

$$+\frac{K}{12}(3\bar{y}^2-2\bar{y}^3)+\frac{R_1K}{120}\Big(\frac{4}{7}\bar{y}^7-2\bar{y}^6+3\bar{y}^5-5\bar{y}^4+10\bar{y}^3-\frac{17}{2}\bar{y}^2$$

$$+R_1^2K(-0{\cdot}0003\bar{y}^{10}+0{\cdot}0004\bar{y}^9+0{\cdot}0016\bar{y}^8-0{\cdot}0074\bar{y}^7$$

$$+0{\cdot}014\bar{y}^6-0{\cdot}0141\bar{y}^5+0{\cdot}0208\bar{y}^4-0{\cdot}0236\bar{y}^3+0{\cdot}006\bar{y}^2)$$

$$+R_2K\Big[\frac{1}{120}\Big(-\frac{4}{7}\bar{y}^7+2\bar{y}^6-3\bar{y}^5+\frac{7}{2}\bar{y}^2\Big) -$$

$$+R_1(-0{\cdot}0001\bar{y}^{11}+0{\cdot}0006\bar{y}^{10}-0{\cdot}0008\bar{y}^9-0{\cdot}0012\bar{y}^8\Big]$$

$$+0{\cdot}0065\bar{y}^7-0{\cdot}0127\bar{y}^6+0{\cdot}0103\bar{y}^5+0{\cdot}0097\bar{y}^3-0{\cdot}0191\bar{y}^2)]$$

$$+R_2^2K(-0{\cdot}0003\bar{y}^{10}+0{\cdot}0004\bar{y}^9-0{\cdot}0004\bar{y}^8+0{\cdot}0009\bar{y}^7$$

$$-0{\cdot}0012\bar{y}^6-0{\cdot}001\bar{y}^5+0{\cdot}0044\bar{y}^2). \quad (48)$$

7. Pressure Distribution

From equations (15) and (16), we have

$$\frac{\partial \bar{p}}{\partial \bar{y}} = (R_1-R_2)f''-(R_1-R_2)^2ff'-R_2(R_1-R_2)f', \qquad \ldots (49)$$

and

$$\frac{\partial \bar{p}}{\partial \bar{x}} = (R_2-R_1)\bar{x}A-K. \qquad \ldots (50)$$

Therefore, on integrating equations (49) and (50), we get

$$\bar{p}(\bar{x}, \bar{y}) = \tfrac{1}{2}(R_2-R_1)\bar{x}^2A-K\bar{x}+(R_1-R_2)f'-\tfrac{1}{2}(R_1-R_2)^2f^2-R_2(R_1-R_2)f+\text{constant}, \qquad \ldots (51)$$

where A is given by (30).

The pressure drop in the $\bar{x}$-direction is

$$P_{\bar{x}} = \bar{p}(0, \bar{y})-\bar{p}(\bar{x}, \bar{y})$$

$$= K\bar{x}+(R_1-R_2)\left[12+\frac{81}{35}(R_1-R_2)\right]\frac{\bar{x}^2}{2}, \qquad \ldots (52)$$

and the pressure drop in $\bar{y}$-direction is

$$P_{\bar{y}} = \bar{p}(\bar{x}, 0)-\bar{p}(\bar{x}, \bar{y})$$

$$= \tfrac{1}{2}(R_1-R_2)^2f^2+R_2(R_1-R_2)f-(R_1-R_2)f'-\tfrac{1}{2}(R_1^2-R_2^2). \qquad \ldots (53)$$

8. Friction Coefficient

The coefficient of skin friction at the wall is

$$c_f = \frac{\tau}{(\mu\nu/b^2)} = \frac{\partial \bar{u}}{\partial \bar{y}} \qquad \ldots (54)$$

From equations (43) and (55), we have

$$(c_f)_{\bar{y}=0} = (R_1-R_2)\bar{x}\left[6-\frac{19}{35}R_1-\frac{16}{35}R_2\right]$$

$$+K\left[\tfrac{1}{2}-\frac{17}{120}R_1+0{\cdot}0121\,R_1^2+\frac{7}{120}R_2-0{\cdot}0383\,R_1R_2+0{\cdot}0087\,R_2^2\right], \qquad \ldots (55)$$

and

$$(c_f)_{\bar{y}=1} = (R_2-R_1)\bar{x}\left[\ 6+\frac{16}{35}R_1+\frac{19}{35}R_2\ \right]$$

$$+K\left[\ -\tfrac{1}{2}+\frac{7}{120}R_1+0{\cdot}436\,R_1^2-\frac{17}{120}R_2+0{\cdot}485\,R_1R_2-0{\cdot}393\,R_2^2\right].$$

9. Numerical Discussion

The stream-lines are shown in figures (1) and (2) for the case of suction at both walls and for suction at the upper wall and injection at the lower wall respectively In figure (1) it is interesting to note that there is back flow for large values of $\bar{x}$ due to the development of adverse pressure gradient as pointed out

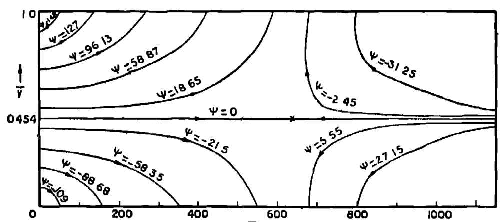

FIG. 1 Streamlines pattern for R_1=-0 18, R_2=0 24 and K=3000

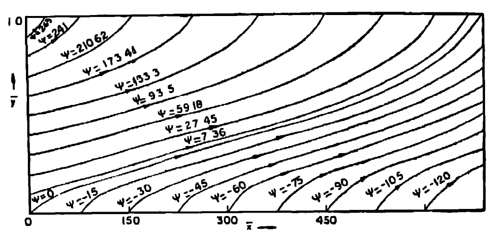

FIG 2 Streamlines pattern for R_1=0 2, R_2=0 4 and K=3000

by Verma & Bansal (1966). As such on the stream-line $\psi = 0$, we obtain a stagnation point for $\bar{x} = 637{\cdot}06$ In figure (3), the axial pressure drop is plotted against $\bar{x}$. The pressure drop has been found to decrease with the increase of R_2 while it increases with the increase of applied pressure gradient The coefficient of friction at both the walls are plotted in figure (4). It is interesting to find that at the lower plate the coefficient of friction increases with the increase of R_1 and R_2 except near the mouth of the channel, whereas at the upper plate the coefficient of friction decreases with the increase of R_1 and R_2.

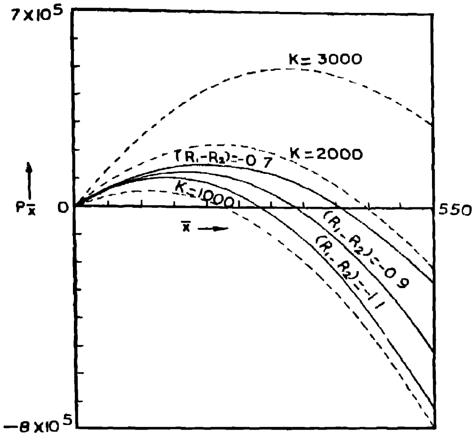

FIG 3 Axial pressure drop against $\bar{x}$

—— for K=1500 and (R_1-R_2) arbitrary.

---- for $(R_1-R_2)=-0.9$ and K arbitrary.

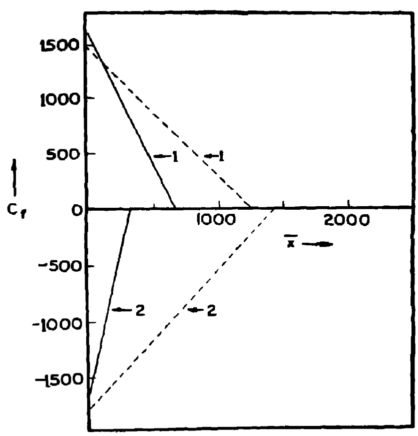

FIG.4 Coefficient of skinfriction against $\bar{x}$

—— for $R_1=-0.18$ and $R_2=0.24$

---- for $R_1=0.2$ and $R_2=0.4$

1 – for $(C_f)_{\bar{y}=0}$ and 2 – for $(C_f)_{\bar{y}=1}$

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